#### Example: associative laws for sets

and

L∩ M∩N

Let The proof of this statement is Theorem: Distribution Laws for SetsL, M and N be sets. Then the following two rules apply:LL leftMM to you for practice. The procedure is very similar to theNN = L M LL NN

previous proof.

The following important rules were written by the British mathematician Augustus de Morgan (1806—1871) and were also named after him. Theorem: Rules of de MorganLet Proof:L, M and N be sets. Then the following two rules apply:L\L\ MM NN == L\ML\M L\NL\N

contained in only one of the two sets then Thus Let We first prove the first rule of de Morgan and proceed analogously to the previous proofs.because of N, then even x ∈ L\ML\(M ∩ N)⊆ (L\M) ∪ (L\N) x ∈ Lx ∈ L\M and therefore , x ∈. Then and is L\Nx ∈ L\Nxx ∈ (L\M) ∪ (L\N) is and therefore ∈ Lfollows. and and thus in particular M, Nx ∉ M ∩ N or in none of them. If x ∈ (L\M) applies accordingly. If . The latter means that is x ∈ (L\M) ∪ (L\N)∪ (L\N)x ∈ M∧ x ∉ N. If x ∉ M∧ x ∈ Nx ∉ Mx is is either applies.∧ x ∉, then, x ∈ L\(M ∩ N)

then x ∈ (L\M) ∪ (L\N)L L\(M ∩ N)⊇ (L\M) ∪ (L\N)

The second rule of de Morgan is proved in an analogous way (we leave this to you for prac-tice). The overall conclusion is therefore that the allegation is correct. □ The following theorem shows some more interesting properties of subsets with respect to union and intersection.